Ministry of Higher Education and Science of the Russian Federation
Ufa State Aviation Technical University

Department of Strength of Materials

METHODOLOGICAL INSTRUCTIONS
for performing
CALCULATION AND GRAPHIC WORK
in the discipline

“FUNDAMENTALS OF MECHANICS, DESIGN” AND MANUFACTURE OF PRODUCTS FROM COMPOSITE MATERIALS"

5 Layer Composite Design

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INTRODUCTION

The fundamentals of mechanics, design and manufacture of products made of composite materials are currently becoming one of the most important professional disciplines studied in higher education. The present and future development of materials science is and will be based on composite materials.

Mechanics of composite materials addresses the most important issues of strength, stiffness, stability and durability of structural elements made of composite materials. Students' assimilation of theoretical material and its application for solving specific practical problems is possible only if they independently complete the calculation homework.

The task consists of two parts. In the first part, the effective elastic characteristics of a unidirectional monolayer are determined using the known characteristics of reinforcing fibers and binders. In the second part, the elastic characteristics of a 5-layer laminate of a given structure are determined using the known characteristics of a unidirectional layer and reinforcement angles.

1. PURPOSE OF THE WORK

The purpose of this work is to acquire practical skills for students to design and determine the elastic characteristics of multilayer composite materials.

2. REQUIREMENTS FOR THE CONTENT AND DESIGN OF THE WORK

PROCEDURE FOR ISSUANCE AND ACCEPTANCE OF CALCULATION WORK

The issuance, consultation and acceptance of work is carried out by a teacher conducting practical classes in a group.

The work code consists of **four digits**. The first two correspond to the structure of the designed CM (Appendix 1, Table 1), the third number – to the type of reinforcing material (Table 2), the fourth number – to the type of matrix (Table 3).

The deadlines for completing homework are determined by the calendar schedule of the educational process.

When accepting an assignment, the teacher must ensure that it is completed by the student independently and in accordance with the requirements specified below.

Accepted assignments are signed by the teacher and stored at the department.

REQUIREMENTS FOR COMPLETING CALCULATION WORK

The completed work is drawn up in the form of a calculation and explanatory note on A4 paper in accordance with GOST 2.105 ESKD “General requirements for text documents”. The **explanatory note** consists of a cover, title page, contents, text part and bibliography.

1. **The title page** is filled out on the front side of the A4 format cover. It indicates in drawing (or printed) font according to GOST 2.304-81: name of the ministry, name of the university, name of the department, type of work (calculation and graphic work, coursework), name of the task, code of the explanatory note, surname, initials of the

student and group code, surname and initials of the teacher, date of submission of the assignment to the teacher, academic year.

2. **The text part** is drawn up on one side of A4 format according to form 2a GOST 2.105-95 (Appendix V) in ink, graphics - in pencil in the text or on separate inserts.

The first page of the text part contains only the conditions of the problems and all data, including design diagrams and other drawings. On the second page is the contents of the work, which lists the titles of all sections of the explanatory note and the corresponding page numbers of the text.

Page numbering must be continuous. The first page is the title page, on which the page number is not affixed. Numbering starts from the third page.

The text part is divided into sections and subsections, which must have headings in strict accordance with the content of the work. Sections must have serial numbers throughout the document and be designated by numbers (for example: 1, 2, etc.). Subsections must be sequentially numbered within each section. For example: 1.1; 1.2; ... - paragraphs of the first section; 1.2.1; 1.2.2; ... - paragraphs of subsection 1.2, etc.

Formulas are numbered with numbers (in brackets) and consist of the section number and the serial number of the formula in the section, separated by a dot.

Calculation formulas are first written in literal expression, then, if necessary, transformations are made in general form, then the alphabetic symbols are replaced with their numerical values and, excluding intermediate actions, the final value of the calculated value is given with the obligatory indication of the dimension. For example,

$$\sigma = \frac{N}{A} = \frac{1 \cdot 10^6}{0,5} = 2 \cdot 10^6 \, Pa \ = \ 2 \, MPa \ . \qquad (1.1)$$

Calculations are usually limited to three significant numbers.

Calculation diagrams and sketches , explaining the pictures and drawings, are located in the text or on separate sheets of paper and are drawn to scale using a ruler and compass.

From start to finish, the text should show the logical connection of the operations being performed.

In the final part of the task, an analysis of the result obtained is required.

The presentation is in the first person in the plural, for example, “there is calculated”, “there is computed”, etc.

3. **List of references:** a list of books and manuals used in performing the work is provided, in accordance with GOST 7.1-84.

4. **Contents** : the titles of all sections of the explanatory note and the corresponding page numbers of the text are listed.

3. METHODOLOGY FOR PERFORMING STANDARD TASKS

TASK TO COMPLETE THE WORK AND CONTENTS OF THE EXPLANATORY NOTE

TASK: Design a multilayer composite material and determine its elastic characteristics in the direction of the axes of the selected coordinate system.

INPUT DATA: Type of reinforcing material, elastic and strength characteristics of threads, reinforcement coefficient, type of matrix and its elastic characteristics, number of unidirectional layers, their thickness and laying angles.

IT'S NECESSARY:

- to determine the elastic and strength characteristics of a unidirectional layer;
- to design a multilayer component in accordance with the specified layer laying angles and reinforcement theory;
- to determine the coefficients of the generalized Hooke's law for a layered composite;
- to determine the elastic characteristics of a multilayer composite in the direction of the selected coordinate axes;

In accordance with the assignment, the explanatory note must contain the following sections:

1. **Initial data**
   - 1.1. **Type of reinforcing material and its elastic and strength characteristics:**
     longitudinal modulus of elasticity E_f, shear modulus G_f, Poisson's ratio ν_f, ultimate elongation $\bar{\varepsilon}_f^+$, tensile strength $\bar{\sigma}_f^+$, reinforcement coefficient ψ_f.
   - 1.2. **Type of matrix material and its elastic characteristics:**
     modulus of elasticity E_m, shear modulus G_m, Poisson's ratio ν_m, ultimate elongation $\bar{\varepsilon}_m^+$.
   - 1.3. **Layer thicknesses** $h_1, h_2, \ldots h_n$ and **layer laying angles** (in rad.) $\varphi_1/\pi, \varphi_2/\pi, \ldots \varphi_n/\pi$.
2. **Determination of elastic and strength characteristics of a unidirectional layer**

2.1. Calculation of the longitudinal modulus of elasticity along the fibers (direction 1).
2.2. Calculation of the transverse (in the laying plane) elastic modulus (direction 2).
2.3. Calculation of shear modulus in plane 1-2.
2.4. Calculation of Poisson's ratio.
2.5. Calculation of the tensile strength of a unidirectional layer in the direction of reinforcement.

3. **Design of a multilayer composite with a given number of layers**
   3.1. The middle surface of the element is selected.
   3.2. The structure and sequence of laying layers is determined in accordance with the theory of reinforcement.
4. **Determination of elastic characteristics of a multilayer composite**
   4.1. Calculation of relative layer thicknesses.
   4.2. Calculation of the coefficients of the generalized Hooke's law for a layered element.
   4.3. Determination of the elastic modulus in the *X-axis direction* of a multilayer element.
   4.4. Determination of the elastic modulus in the *Y-axis direction* .
   4.5. Determination of shear modulus in the *XY plane* .
   4.6. Determination of Poisson's ratios ν_{xy}, ν_{yx}.
   4.7. Conclusions.
5. **Bibliography**

METHODOLOGY FOR PERFORMING A TYPICAL TASK

Design a five-layer composite and determine its elastic characteristics.

INPUT DATA:

1. **Layer thicknesses** h_1, h_2, h_3, h_4, h_5.
2. **Layer laying angles** φ_1/π, φ_2/π, φ_3/π, φ_4/π, φ_5/π.
3. **Type of reinforcing thread and its mechanical characteristics** (for example, glass fiber):
   E_f - modulus of elasticity of the reinforcing thread,
   G_f - shear modulus,
   ν_f - Poisson's ratio,

$\overline{\varepsilon}_f^+$ - maximum relative elongation of the thread when stretched,

$\overline{\sigma}_f^+$ - tensile strength of the thread,

ψ_f - reinforcement coefficient.

4. **Type of matrix (for example, epoxy) and its mechanical characteristics**:

   E_m - tensile modulus of elasticity,

   G_m - shear modulus,

   ν_m - Poisson's ratio.

5. **The elastic and strength characteristics of the unidirectional layer are determined**

5.1. Modulus of elasticity in the direction of reinforcement:

$$E_1 = E_f \psi_f + E_m (1 - \psi_f);$$

5.2. Transverse modulus of elasticity E_2

$$\frac{1}{E_2} = \frac{\psi_f}{E_f} + \frac{1 - \psi_f}{E_m};$$

5.3. Shear modulus G_{12}

$$\frac{1}{G_{12}} = \frac{\psi_f}{G_f} + \frac{1 - \psi_f}{G_m};$$

5.4. Poisson's ratio

$$\nu_{12} = \nu_f \psi_f + \nu_m (1 - \psi_f)$$

(ν_{12} - the first index indicates the direction of stress, the second – the direction of transverse deformation);

5.5. Tensile strength in the direction of reinforcement

$$\overline{\sigma}_1^+ = \overline{\sigma}_f^+ (\psi_f + \frac{E_m}{E_f}(1 - \psi_f)).$$

DESIGN OF A FIVE-LAYER COMPOSITE

1. We assume that the **structure** of the composite is **symmetrical** in thickness relative to the middle surface (layer) both in terms of the angle of laying the layers and in thickness, i.e. a layer with a laying angle φ_1 and thickness h_1 located on one side of the middle layer must correspond to a layer with the same laying angle φ_1 and thickness h_1 located on the other side of the middle surface at the same distance from it (Fig. 1).

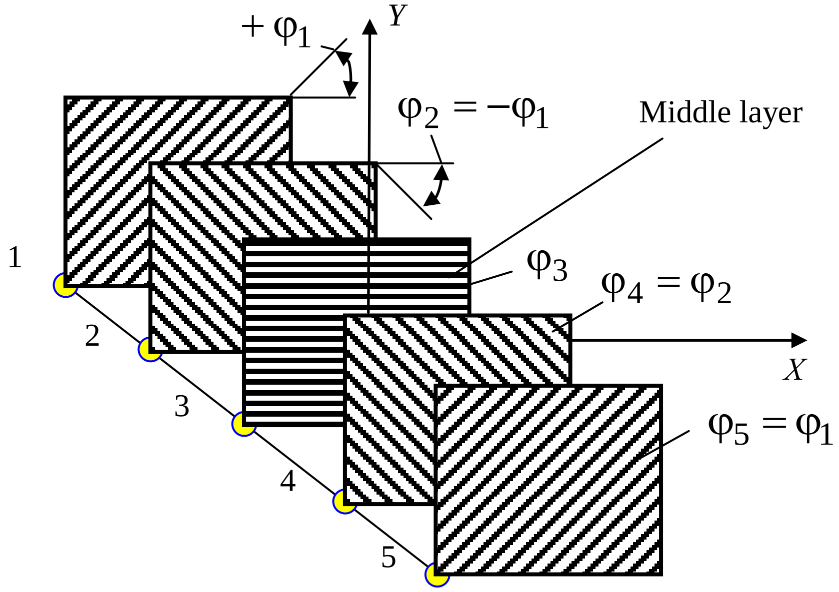

Fig. 1 - Structure of a five-layer composite by thickness

2. The structure of the composite must satisfy **the second requirement**: each layer with a laying angle + φ_i must correspond to a layer on the other side of the middle layer with a laying angle -φ_i, located at the same distance from the middle layer.

   These two distance requirements cannot be satisfied. As a rule, the symmetry of the laying is satisfied first.

DETERMINATION OF ELASTIC CHARACTERISTICS OF MULTILAYER COMPOSITE

• Calculation of relative layer thicknesses:

$$\bar{h}_1 = \frac{h_1}{h_1 + h_2 + h_3 + h_4 + h_5}, \qquad \bar{h}_2 = \frac{h_2}{h_1 + h_2 + h_3 + h_4 + h_5},$$

$$\bar{h}_3 = \frac{h_3}{h_1 + h_2 + h_3 + h_4 + h_5}, \qquad \bar{h}_4 = \frac{h_4}{h_1 + h_2 + h_3 + h_4 + h_5},$$

$$\bar{h}_5 = \frac{h_5}{h_1 + h_2 + h_3 + h_4 + h_5}.$$

• Calculation of the coefficients of the generalized Hooke's law for a five-layer element:

$$B_{11} = \sum_{i=1}^{5} \bar{h}_i [\frac{E_1^i}{1 - \nu_{12}^i \nu_{21}^i} \cos^4 \varphi_i + 2 \frac{E_1^i \nu_{21}^i}{1 - \nu_{12}^i \nu_{21}^i} \sin^2 \varphi_i \cos^2 \varphi_i +$$

$$+ \frac{E_2^i}{1 - \nu_{12}^i \nu_{21}^i} \sin^4 \varphi_i + G_{12}^i \sin^2 2\varphi_i];$$

$$B_{12} = \sum_{i=1}^{5} \bar{h}_i [\frac{E_1^i + E_2^i}{1 - \nu_{12}^i \nu_{21}^i} \sin^2 \varphi_i \cos^2 \varphi_i + \frac{E_1^i \nu_{21}^i}{1 - \nu_{12}^i \nu_{21}^i} (\sin^4 \varphi_i + \cos^4 \varphi_i) -$$

$$- G_{12}^i \sin^2 2\varphi_i];$$

$$B_{22} = \sum_{i=1}^{5} \bar{h}_i [\frac{E_1^i}{1 - \nu_{12}^i \nu_{21}^i} \sin^4 \varphi_i + 2 \frac{E_1^i \nu_{21}^i}{1 - \nu_{12}^i \nu_{21}^i} \sin^2 \varphi_i \cos^2 \varphi_i +$$

$$+ \frac{E_2^i}{1 - \nu_{12}^i \nu_{21}^i} \cos^4 \varphi_i + G_{12}^i \sin^2 2\varphi_i];$$

$$B_{33} = \sum_{i=1}^{5} \bar{h}_i [\frac{E_1^i + E_2^i - 2E_1^i \nu_{21}^i}{1 - \nu_{12}^i \nu_{21}^i} \sin^2 \varphi_i \cos^2 \varphi_i + G_{12}^i \cos^2 2\varphi_i].$$

• Determination ν_{21}^i from the orthotropic condition:

$$\frac{\nu_{21}^{i}}{E_2^{i}} = \frac{\nu_{12}^{i}}{E_1^{i}}; \ \nu_{21}^{i} = \frac{\nu_{12}^{i} E_2^{i}}{E_1^{i}}.$$

• Determination of elastic characteristics of a five-layer composite:

$$E_x = B_{11} - \frac{B_{12}^2}{B_{22}}; \quad E_y = B_{22} - \frac{B_{12}^2}{B_{11}}; \quad G_{xy} = B_{33};$$

(E_x - modulus of elasticity in the direction of the axis X,
E_y - modulus of elasticity in the direction of the axis Y,
G_{xy} - in-plane shear modulus XY).

• Find Poisson's ratios ν_{xy}, ν_{yx}:

$$\nu_{xy} = \frac{B_{12}}{B_{22}}, \quad \nu_{yx} = \frac{B_{12}}{B_{11}}.$$

CONCLUSION: the designed composite has the following elastic characteristics:

E_x= ;

E_y= ;

G_{xy}= ;

Poisson's ratios:

ν_{xy}= ;

ν_{yx}= .

APPENDIX 1

Table 1

No. layer	Thickness (mm) and laying angle (rad) relative to the X axis	Options														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	h_1 (mm)	0.3	0.25	0.2	0.35	0.4	0.45	0.5	0.6	0.7	0.8	0.9	1	0.25	0.3	0.4
	φ_1/π	1/3	1/4	1/5	1/6	1/9	1/12	1/4	2/9	5/18	7/18	4/9	4/9	1/3	1/5	1/9
2	h_2 (mm)	0.3	0.25	0.2	0.35	0.4	0.45	0.5	0.6	0.7	0.8	0.9	1	0.3	0.2	0.3
	φ_2/π	-1/3	-1/4	-1/5	-1/6	-1/9	-1/12	-1/4	-2/9	-5/18	-7/18	-4/9	-4/9	-1/3	-1/5	-1/9
3	h_3 (mm)	0.3	0.25	0.2	0.35	0.4	0.45	0.5	0.6	0.7	0.8	0.9	1	0.5	0.4	0.3
	φ_3/π	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	h_4 (mm)	0.3	0.25	0.2	0.35	0.4	0.45	0.5	0.6	0.7	0.8	0.9	1	0.3	0.2	0.3
	φ_4/π	-1/3	-1/4	-1/5	-1/6	-1/9	-1/12	-1/4	-2/9	-5/18	-7/18	-4/9	-4/9	-1/3	-1/5	-1/9
5	h_5 (mm)	0.3	0.25	0.2	0.35	0.4	0.45	0.5	0.6	0.7	0.8	0.9	1	0.25	0.3	0.4
	φ_5/π	1/3	1/4	1/5	1/6	1/9	1/12	1/4	2/9	5/18	7/18	4/9	4/9	1/3	1/5	1/9

Continuation of the Table. 1

No. layer	Thickness (mm) and laying angle (rad) relative to the X axis	Options														
		16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	h_1 (mm)	0.4	0.3	0.35	0.5	0.25	0.4	0.5	0.5	0.3	0.6	0.7	0.5	0.4	0.6	0.7
	φ_1/π	1/3	1/4	1/5	1/6	1/12	2/9	1/12	5/12	5/18	7/18	4/9	1/3	1/5	1/9	7/18
2	h_2 (mm)	0.2	0.2	0.25	0.4	0.3	0.3	0.4	0.4	0.5	0.4	0.5	0.3	0.25	0.5	0.5
	φ_2/π	-1/3	-1/4	-1/5	-1/6	-1/12	-2/9	-1/12	-5/12	-5/18	-7/18	-4/9	-1/3	-1/5	-1/9	-7/18
3	h_3 (mm)	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.35	0.45	0.25	0.65	0.85	0.75	0.85
	φ_3/π	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2
4	h_4 (mm)	0.2	0.2	0.25	0.4	0.3	0.3	0.4	0.4	0.5	0.4	0.5	0.3	0.25	0.5	0.5
	φ_4/π	-1/3	-1/4	-1/5	-1/6	-1/12	-2/9	-1/12	-5/12	-5/18	-7/18	-4/9	-1/3	-1/5	-1/9	-7/18
5	h_5 (mm)	0.4	0.3	0.35	0.5	0.25	0.4	0.5	0.5	0.3	0.6	0.7	0.5	0.4	0.6	0.7
	φ_5/π	1/3	1/4	1/5	1/6	1/12	2/9	1/12	5/12	5/18	7/18	4/9	1/3	1/5	1/9	7/18

Table 2

Elastic and strength characteristics of reinforcing fibers

Elastic and strength characteristics	Option			
	1	2	3	4
	Glass fiber	Carbon fiber	Organic fiber	Boron fiber
E_f , GPa	70	250	131	300-400
G_f , GPa	24	12	20	100
ν_f	0.22	0.15	0.25	0.11
Ultimate tensile strength, $\sigma_{u,f}$, MPa	2500	2400-3500	2000-2500	3000-3500
Ultimate relative elongation, %	3.0-3.5	1.0	3-4	≤1.0

Table 3

Elastic and strength characteristics of matrices

Elastic and strength characteristics	Option			
	1	2	3	4
	Polyester binder	Epoxy binder	Epoxyphenol binder	Phenol-formaldehyde binder
E_m, GPa	2.1-4.6	2.8-4.2	2.8-4.1	2.8-4.6
G_m, GPa	1.0-1.9	0.8-1.5	1.1-1.6	1.0-1.4
ν_m	0.35-0.42	0.34-0.4	0.33-0.4	0.35
Ultimate tensile strength, $\sigma_{u,m}$, MPa	42-70	28-91	33-86	42-63
Ultimate relative elongation, %	6	2-6	1.8-3.2	1.5-2.0

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1. Yu.S. Pervushin, V.S. Zhernakov . Fundamentals of design and manufacturing technology of products from layered composite materials: textbook. allowance / Ufimsk . state aviation tech . univ. – Ufa: USATU, 2008 – 201 pp.

2. Yu.S. Pervushin, V.S. Zhernakov . Design and prediction of mechanical properties of a unidirectional layer of composite materials: textbook. allowance / Ufimsk . state aviation tech . univ. – Ufa: USATU, 2002 – 127 pp.